Supplementary material for

What do beta diversity components reveal from presence-absence community data? Let us connect every indicator to an indicandum!

by

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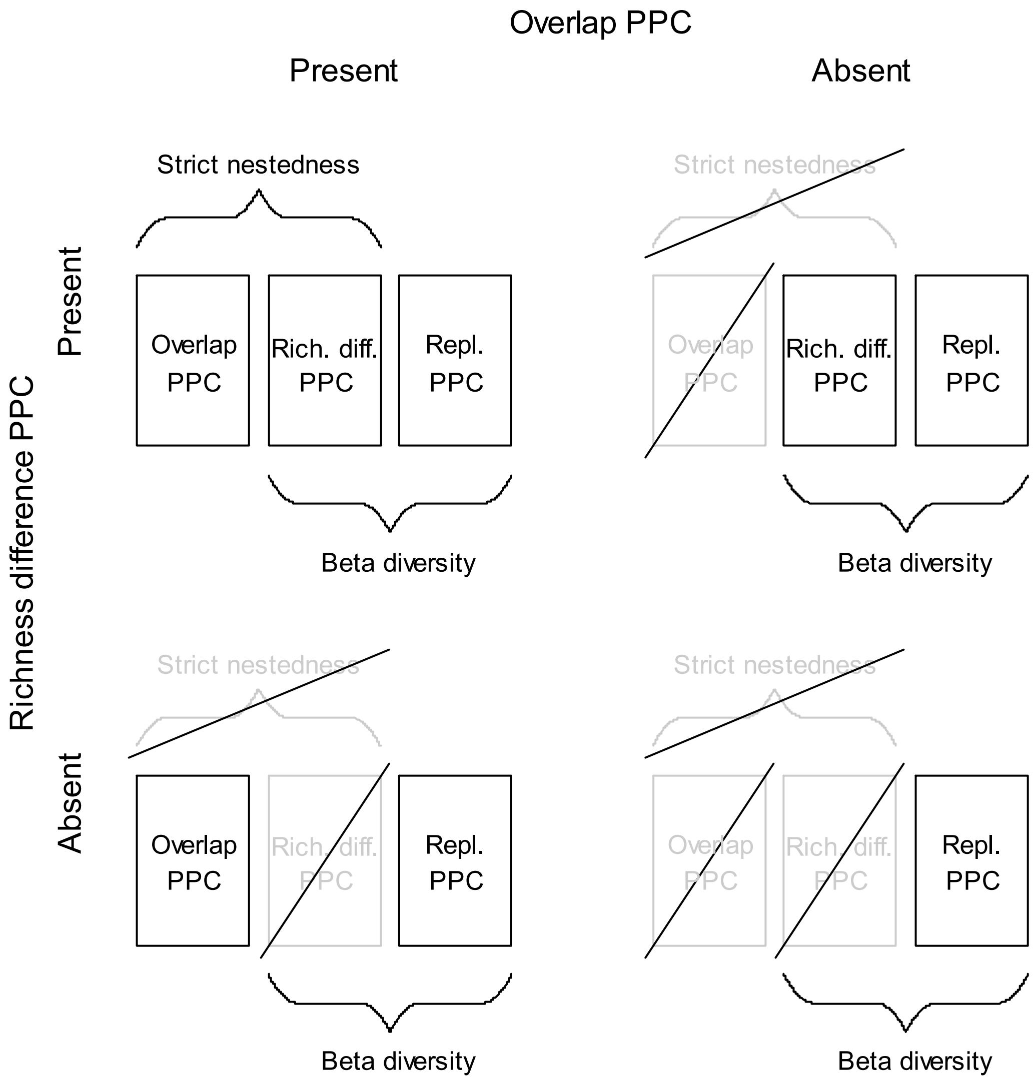
This supplementary material complements the main paper by explaining under which circumstances the Richness diffence Pairwise Pattern Component (*PPC*) can be interpreted as the nestedness-related component of beta diversity. Intepretation depends on the presence and absence of Overlap and Richness difference *PPC*s, when nestedness is defined in the strict sense (i.e. nestedness is defined such that species of one site is proper subset of those in the other). Supplementary Fig. 1 shows that Richness difference *PPC* forms the nestedness-related component of beta diversity only when both Overlap and Richness difference *PPC* exist. When Richness difference *PPC* is present but Overlap *PPC* is not, then nestedness does not exist and thus the Richness difference *PPC* cannot be interpreted as the nestedness-related component of beta diversity. If Richness difference PPC is absent, then Richness difference *PPC* cannot be the nestedness related component of beta diversity because neither the *PPC* nor nestedness exists.

Supplementary Table 1 summarizes the existing and new measures quantifying *PPC*s and derived concepts. The table shows that these measures are indicators of the ecological concepts formulated by *PPC*s in any presence-absence based community pattern occuring in nature. Supplementary Table 2 summarizes the existing methods of beta diversity partitioning via examining pairs of sites based on presence-absence data.

The three other tables show how replacement (Suppl. Table 3), nestedness-related (Suppl. Table 4) and richness difference (Suppl. Table 5) components of beta diversity partitions satisfy the properties defined in the main document.

Finally, Supplementary Appendix 1 provides an R sript for the calculation of the intersection of nestedness and beta diversity (*I*) and the relative complement of nestedness in beta diversity (*RC*).



Supplementary Fig. 1: A schematic representation of how Richness Difference *PPC* can be interpreted as the nestedness-related fraction of beta diversity depending on the presence and absence of Overlap and Richness difference *PPC*s, when nestedness is interpreted in the strict sense (Strict nestedness). Rich. diff. = Richness difference, Repl. = Replacement. Note that this figure differs from Fig. 2 of the main document in that here nestedness is interpreted as strict nestedness (while in Fig. 2 as Broad nestedness). The difference manifested when Overlap *PPC* is present and Richness difference *PPC* is absent (left bottom subfigures). In broad sense nestedness definition (Fig. 2 of the main document) both nestedness and beta diversity concepts exist but Richness difference *PPC* cannot be their common fraction because it does not exist. In strict sense nestedness definition (this figure), Richness difference *PPC* cannot be the common fraction of (strict) nestedness and beta diversity because Richness difference *PPC*, as well as the concept of (strict) nestedness do not exist.

Supplementary Table 1: The different measurement systems to quantify the size of the *PPC*s and derived concepts. The *a*, *b* and *c* parameters refer to the 2x2 contingency table, see main text.

	The *PPC*s is measured by			
	the number of species		the number of presences	
		expressed as		
	raw number	relativized number[1]	raw number	relativized number[2]
PPC	SDR_{WB}[3]	SDR_J[4]	SDR_{pres}[5]	SDR_S[6]
Overlap	a	$\frac{a}{a+b+c}$	$2a$	$\frac{2a}{2a+b+c}$
Replacement	$2\min(b,c)$	$\frac{2\min(b,c)}{a+b+c}$	$2\min(b,c)$	$\frac{2\min(b,c)}{2a+b+c}$
Richness difference	$\lvert b-c\rvert$	$\frac{\lvert b-c\rvert}{a+b+c}$	$\lvert b-c\rvert$	$\frac{\lvert b-c\rvert}{2a+b+c}$
Overlap and Replacement together (=richness agreement)	$a + 2\min(b,c)$	$\frac{a+2\min(b,c)}{a+b+c}$	$2a + 2\min(b,c)$	$\frac{2a+2\min(b,c)}{2a+b+c}$
Replacement and Richness difference together (=beta diversity)	$b+c$	$\frac{b+c}{a+b+c}$	$b+c$	$\frac{b+c}{2a+b+c}$
Overlap and Richness difference together with the condition that Overlap *PPC* exists (=broad sense nestedness)	a + |b-c| if a>0, otherwise 0	$\frac{a+\lvert b-c\rvert}{a+b+c}$ if a>0, otherwise 0	2a + |b-c| if a>0, otherwise 0	$\frac{2a+\lvert b-c\rvert}{2a+b+c}$ if a>0, otherwise 0
Overlap with Richness difference together with the condition that both Overlap and Richness difference *PPC*s exist (=strict sense nestedness)	a + |b-c| if a>0 and |b-c|>0, otherwise 0	$\frac{a+\lvert b-c\rvert}{a+b+c}$ if a>0 and |b-c|>0, otherwise 0	2a + |b-c| if a>0 and |b-c|>0, otherwise 0	$\frac{2a+\lvert b-c\rvert}{2a+b+c}$ if a>0 and |b-c|>0, otherwise 0

[1]Relativization is made by the total number of species present in both sites

[2]Relativization is made by the total number of presences in both sites

[3]In Podani and Schmera (2011), SDR was written without a subscript. But since Weiher and Boylen (1994) had defined beta diversity of pairs of sites as *b*+*c* (see also Koleff et al., 2003), Podani and Schmera (2016) did use the subscript WB when referring to this simplex.

[4]Subscript J refers to Jaccard, see Podani and Schmera (2016)

[5]Subscript pres refers to the number of presences. This is a new suggestion.

[6]Subscript S refers to the Sørensen index. The idea was seeded in Carvalho et al. (2013) and Legendre (2014) but is fully expanded here.

Supplementary Table 2: Overview of existing methods of beta diversity partitioning by examination of pairs of sites based on presence-absence data.

Family	Framework	Measure	Abbr.	Formula	References
Weiher-Boylen (WB)		Weiher-Boylen diversity	β_{WB}	*b+c*	Weiher and Boylen (1994), Koleff et al. (2003)
	POD	replacement	$Repl_{WB}$	*2min(b.c)*	Podani and Schmera (2011)
		richness difference	$Rich_{WB}$	*|b-c|*	Podani and Schmera (2011)
	SET	intersection of nestedness and (Weiher-Boylen) beta diversity	I_{WB}	*|b-c| if a>0 otherwise 0*	this paper
		relative complement of nestedness in (Weiher-Boylen) beta diversity	RC_{WB}	*2min(b,c) if a> 0 otherwise b+c*	this paper
Jaccard (J)		Jaccard dissimilarity	D_J	$\frac{b+c}{a+b+c}$	Jaccard (1912)
	BAS	replacement	$Repl_{BJ}$	$\frac{2\min(b,c)}{a+2\min(b,c)}$	Baselga (2012)
		nestedness resultant component	Nes_{BJ}	$\frac{\|b-c\|}{a+b+c}\times\frac{a}{a+2\min(b,c)}$	Baselga (2012)
	POD	replacement	$Repl_J$	$\frac{2\min(b,c)}{a+b+c}$	Cardoso et al. (2009), Podani and Schmera (2011)
		richness difference	$Rich_J$	$\frac{\|b-c\|}{a+b+c}$	Podani and Schmera (2011)
	SET	intersection of nestedness and (Jaccard) beta diversity	I_J	$\frac{\|b-c\|}{a+b+c}$ *if a>0 otherwise 0*	this paper
		relative complement of nestedness in (Jaccard) beta diversity	RC_J	$\frac{2\min(b,c)}{a+b+c}$ *if a>0 otherwise* $\frac{b+c}{a+b+c}$	this paper
Sørensen (S)		Sørensen dissimilarity	D_S	$\frac{b+c}{2a+b+c}$	Sørensen (1948)
	BAS	replacement (turnover) = Simpson dissimilarity	$Repl_{BS}$	$\frac{\min(b,c)}{a+\min(b,c)}$	Simpson (1943), Baselga 2010
		nestedness resulted component	Nes_{BS}	$\frac{\|b-c\|}{2a+b+c}\times\frac{a}{a+\min(b,c)}$	Baselga (2010)
	POD	replacement	$Repl_S$	$\frac{2\min(b,c)}{2a+b+c}$	Legendre (2014), Baselga and Leprieur (2015)
		richness difference	$Rich_S$	$\frac{\|b-c\|}{2a+b+c}$	Legendre (2014), Baselga and Leprieur (2015)
	SET	intersection of nestedness and (Sørensen) beta diversity	I_S	$\frac{\|b-c\|}{2a+b+c}$ *if a>0 otherwise 0*	this paper
		relative complement of nestedness in (Sørensen) beta diversity	RC_S	$\frac{2\min(b,c)}{2a+b+c}$ *if a>0 otherwise* $\frac{b+c}{2a+b+c}$	this paper

Supplementary Table 3: Overview of the performance of the different replacement components of beta diversity in detecting Replacement *PPC* in different community patterns. We examined the replacement components of the Weiher-Boylen (WB), Jaccard (Jac) and Sørensen (Sør) families in the POD and BAS frameworks (see Suppl. Table 2, for more details). We examined overlap, replacement, perfect richness agreement, perfect nested anti-nested and perfect complex patterns (see Table 1 as well in the main document for further explanation). PRO 1 to PRO 5 are properties of beta diversity components (see main document). To support comparison of beta diversity and replacement components, their algebraic forms are given in the "beta diversity" and "replacement" columns. Cells below PRO 1 to PRO 5 indicate whether the replacement component possesses (Yes) or not (No) the particular property (column) regarding the given community pattern (row).

Family	Framework	Community pattern	Beta diversity	Replacement	PRO 1	PRO 2	PRO 3	PRO 4	PRO 5
WB	POD	overlap	0	0	Yes	Yes	Yes	No	No
		replacement	2min(b,c)	2min(b,c)	Yes	Yes	Yes	No	No
		perfect richness agreement	2min(b,c)	2min(b,c)	Yes	Yes	Yes	No	No
		perfect nested	|b-c|	0	Yes	Yes	Yes	No	No
		anti-nested	b+c	2min(b,c)	Yes	Yes	Yes	No	No
		perfect complex	b+c	2min(b,c)	Yes	Yes	Yes	No	No
Jac	BAS	overlap	$\frac{0}{a}=0$	$\frac{0}{a}=0$	Yes	Yes	Yes	No	No
		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{2\min(b,c)}{2\min(b,c)}=1$	Yes	Yes	No	Yes	Yes
		perfect richness agreement	$\frac{2\min(b,c)}{a+2\min(b,c)}$	$\frac{2\min(b,c)}{a+2\min(b,c)}$	Yes	Yes	No	Yes	No
		perfect nested	$\frac{\lvert b-c\rvert}{a+\lvert b-c\rvert}$	$\frac{0}{a}=0$	Yes	Yes	No	No	No
		anti-nested	$\frac{b+c}{b+c}=1$	$\frac{2\min(b,c)}{2\min(b,c)}=1$	Yes	No	No	No	No
		perfect complex	$\frac{b+c}{a+b+c}$	$\frac{2\min(b,c)}{a+2\min(b,c)}$	Yes	Yes	No	No	No
	POD	overlap	$\frac{0}{a}=0$	$\frac{0}{a}=0$	Yes	Yes	No	Yes	No

		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{2\min(b,c)}{2\min(b,c)}=1$	Yes	Yes	No	Yes	Yes
		perfect richness agreement	$\frac{2\min(b,c)}{a+2\min(b,c)}$	$\frac{2\min(b,c)}{a+2\min(b,c)}$	Yes	Yes	No	Yes	No
		perfect nested	$\frac{\|b-c\|}{a+\|b-c\|}$	$\frac{0}{a+\|b-c\|}=0$	Yes	Yes	Yes	Yes	No
		anti-nested	$\frac{b+c}{b+c}=1$	$\frac{2\min(b,c)}{b+c}$	Yes	Yes	No	Yes	Yes
		perfect complex	$\frac{b+c}{a+b+c}$	$\frac{2\min(b,c)}{a+b+c}$	Yes	Yes	No	Yes	No
Sør	BAS	overlap	$\frac{0}{2a}=0$	$\frac{0}{2a}=0$	Yes	Yes	No	No	Yes
		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{2\min(b,c)}{2\min(b,c)}=1$	Yes	Yes	No	Yes	Yes
		perfect richness agreement	$\frac{2\min(b,c)}{2a+2\min(b,c)}$	$\frac{2\min(b,c)}{2a+2\min(b,c)}$	Yes	Yes	No	No	Yes
		perfect nested	$\frac{\|b-c\|}{2a+\|b-c\|}$	$\frac{0}{2a+\|b-c\|}=0$	Yes	Yes	No	No	Yes
		anti-nested	$\frac{b+c}{b+c}=1$	$\frac{2\min(b,c)}{2\min(b,c)}=1$	Yes	No	No	No	No
		perfect complex	$\frac{b+c}{2a+b+c}$	$\frac{2\min(b,c)}{2a+2\min(b,c)}$	Yes	Yes	No	No	Yes
	POD	overlap	$\frac{0}{2a}=0$	$\frac{0}{2a}=0$	Yes	Yes	No	No	Yes
		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{2\min(b,c)}{2\min(b,c)}=1$	Yes	Yes	No	Yes	Yes
		perfect richness agreement	$\frac{2\min(b,c)}{2a+2\min(b,c)}$	$\frac{2\min(b,c)}{2a+2\min(b,c)}$	Yes	Yes	No	No	Yes

perfect nested	$\frac{\lvert b-c \rvert}{2a+\lvert b-c \rvert}$	$\frac{0}{2a+\lvert b-c \rvert}=0$	Yes	Yes	No	No	Yes
anti-nested	$\frac{b+c}{b+c}=1$	$\frac{2\min(b,c)}{b+c}$	Yes	Yes	No	Yes	Yes
perfect complex	$\frac{b+c}{2a+b+c}$	$\frac{2\min(b,c)}{2a+b+c}$	Yes	Yes	No	No	Yes

Supplementary Table 4: Overview of the performance of the different nestedness-related components of beta diversity in detecting the nestedness-related PPC of beta diversity (i.e. Richness difference *PPC* with some conditions applied) in different community patterns. We examined the replacement components of the Weiher-Boylen (WB), Jaccard (Jac) and Sørensen (Sør) families in the SET and BAS frameworks (see Suppl. Table 2, for more details). We examined overlap, replacement, perfect richness agreement, perfect nested anti-nested and perfect complex patterns (see Table 1 as well in the main document for further explanation). PRO 1 to PRO 5 are properties of beta diversity components (see main document). To support comparison of beta diversity and nestedness-related components, their algebraic forms are given in the "beta diversity" and "nestedness-related" columns. Cells below PRO 1 to PRO 5 indicate whether the nestedness-related component possesses (Yes) or not (No) the particular property (column) regarding the given community pattern (row).

Family	Framework	Community pattern	Beta diversity	Nestedness-related component	PRO 1	PRO 2	PRO 3	PRO 4	PRO 5
WB	SET	overlap	0	0	Yes	Yes	Yes	No	No
		replacement	2min(b,c)	0	Yes	Yes	Yes	No	No
		perfect richness agreement	2min(b,c)	0	Yes	Yes	Yes	No	No
		perfect nested	|b-c|	|b-c|	Yes	Yes	Yes	No	No
		anti-nested	b+c	0	Yes	Yes	Yes	No	No
		perfect complex	b+c	|b-c|	Yes	Yes	Yes	No	No
Jac	BAS	overlap	$\frac{0}{a}=0$	$\frac{0}{a}\times\frac{a}{a}=0$	Yes	Yes	No	Yes	No
		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{0}{2\min(b,c)}\times\frac{0}{2\min(b,c)}=0$	Yes	Yes	No	No	No
		perfect richness agreement	$\frac{2\min(b,c)}{a+2\min(b,c)}$	$\frac{0}{a+2\min(b,c)}\times\frac{a}{a+2\min(b,c)}=0$	Yes	Yes	No	No	No
		perfect nested	$\frac{\|b-c\|}{a+\|b-c\|}$	$\frac{\|b-c\|}{a+\|b-c\|}\times\frac{a}{a}$	Yes	Yes	No	No	No
		anti-nested	$\frac{b+c}{b+c}=1$	$\frac{\|b-c\|}{b+c}\times\frac{0}{2\min(b,c)}=0$	Yes	Yes	No	No	No
		perfect complex	$\frac{b+c}{a+b+c}$	$\frac{\|b-c\|}{a+b+c}\times\frac{a}{a+2\min(b,c)}$	Yes	Yes	No	No	No

	SET	overlap	$\frac{0}{a}=0$	$\frac{0}{a}=0$	Yes	Yes	No	Yes	No
		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{0}{2\min(b,c)}=0$	Yes	Yes	No	Yes	Yes
		perfect richness agreement	$\frac{2\min(b,c)}{a+2\min(b,c)}$	$\frac{0}{a+2\min(b,c)}=0$	Yes	Yes	No	Yes	No
		perfect nested	$\frac{\lvert b-c\rvert}{a+\lvert b-c\rvert}$	$\frac{\lvert b-c\rvert}{a+\lvert b-c\rvert}$	Yes	Yes	No	Yes	No
		anti-nested	$\frac{b+c}{b+c}=1$	$\frac{0}{b+c}=0$	Yes	Yes	No	Yes	Yes
		perfect complex	$\frac{b+c}{a+b+c}$	$\frac{\lvert b-c\rvert}{a+b+c}$	Yes	Yes	No	Yes	No
Sør	BAS	overlap	$\frac{0}{2a}=0$	$\frac{0}{2a}\times\frac{a}{a}=0$	Yes	Yes	No	No	Yes
		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{0}{2\min(b,c)}\times\frac{0}{2\min(b,c)}=0$	Yes	Yes	No	No	No
		perfect richness agreement	$\frac{2\min(b,c)}{2a+2\min(b,c)}$	$\frac{0}{2a+2\min(b,c)}\times\frac{a}{a+2\min(b,c)}=0$	Yes	Yes	No	No	No
		perfect nested	$\frac{\lvert b-c\rvert}{2a+\lvert b-c\rvert}$	$\frac{\lvert b-c\rvert}{2a+\lvert b-c\rvert}\times\frac{a}{a}$	Yes	Yes	No	No	Yes
		anti-nested	$\frac{b+c}{b+c}=1$	$\frac{\lvert b-c\rvert}{b+c}\times\frac{0}{2\min(b,c)}=0$	Yes	Yes	No	No	No
		perfect complex	$\frac{b+c}{2a+b+c}$	$\frac{\lvert b-c\rvert}{2a+b+c}\times\frac{a}{a+2\min(b,c)}$	Yes	Yes	No	No	No
	SET	overlap	$\frac{0}{2a}=0$	$\frac{0}{2a}=0$	Yes	Yes	No	No	Yes
		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{0}{2\min(b,c)}=0$	Yes	Yes	No	Yes	Yes

	perfect richness agreement	$\frac{2\min(b,c)}{2a+2\min(b,c)}$	$\frac{0}{2a+2\min(b,c)}=0$	Yes	Yes	No	No	Yes
	perfect nested	$\frac{\lvert b-c\rvert}{2a+\lvert b-c\rvert}$	$\frac{\lvert b-c\rvert}{2a+\lvert b-c\rvert}$	Yes	Yes	No	No	Yes
	anti-nested	$\frac{b+c}{b+c}=1$	$\frac{0}{b+c}=0$	Yes	Yes	No	Yes	Yes
	perfect complex	$\frac{b+c}{2a+b+c}$	$\frac{\lvert b-c\rvert}{2a+b+c}$	Yes	Yes	No	No	Yes

Supplementary Table 5: Overview of the performance of the different forms of beta diversity and richness difference components in detecting Richness difference *PPC* in different community patterns. We examined the richness difference components of the Weiher-Boylen (WB), Jaccard (Jac) and Sørensen (Sør) families in in the POD framework (see Suppl. Table 2, for more details). We examined overlap, replacement, perfect richness agreement, perfect nested anti-nested and perfect complex patterns (see Table 1 as well in the main document for further explanation). PRO 1 to PRO 5 are properties of beta diversity components (see main document). To support comparison of beta diversity and richness difference components, their algebraic forms are given in the "beta diversity" and "richness difference" columns. Cells below PRO 1 to PRO 5 indicate whether the richness difference component possesses (Yes) or not (No) the particular property (column) regarding the given community pattern (row).

Family	Framework	Community pattern	Beta diversity	Richness difference	PRO 1	PRO 2	PRO 3	PRO 4	PRO 5
WB	POD	overlap	0	0	Yes	Yes	Yes	No	No
		replacement	2min(b,c)	0	Yes	Yes	Yes	No	No
		perfect richness agreement	2min(b,c)	0	Yes	Yes	Yes	No	No
		perfect nested	|b-c|	|b-c|	Yes	Yes	Yes	No	No
		anti-nested	b+c	|b-c|	Yes	Yes	Yes	No	No
		perfect complex	b+c	|b-c|	Yes	Yes	Yes	No	No
Jac	POD	overlap	$\frac{0}{a}=0$	$\frac{0}{a}=0$	Yes	Yes	No	Yes	No
		replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{0}{2\min(b,c)}=0$	Yes	Yes	No	Yes	Yes
		perfect richness agreement	$\frac{2\min(b,c)}{a+2\min(b,c)}$	$\frac{0}{a+2\min(b,c)}=0$	Yes	Yes	No	Yes	No
		perfect nested	$\frac{\|b-c\|}{a+\|b-c\|}$	$\frac{\|b-c\|}{a+\|b-c\|}$	Yes	Yes	No	Yes	No
		anti-nested	$\frac{b+c}{b+c}=1$	$\frac{\|b-c\|}{b+c}$	Yes	Yes	No	Yes	Yes
		perfect complex	$\frac{b+c}{a+b+c}$	$\frac{\|b-c\|}{a+b+c}$	Yes	Yes	No	Yes	No
Sør	POD	overlap	$\frac{0}{2a}=0$	$\frac{0}{2a}=0$	Yes	Yes	No	No	Yes

replacement	$\frac{2\min(b,c)}{2\min(b,c)}=1$	$\frac{0}{2\min(b,c)}=0$	Yes	Yes	No	Yes	Yes
perfect richness agreement	$\frac{2\min(b,c)}{2a+2\min(b,c)}$	$\frac{0}{2a+2\min(b,c)}=0$	Yes	Yes	No	No	Yes
perfect nested	$\frac{\lvert b-c\rvert}{2a+\lvert b-c\rvert}$	$\frac{\lvert b-c\rvert}{2a+\lvert b-c\rvert}$	Yes	Yes	No	No	Yes
anti-nested	$\frac{b+c}{b+c}=1$	$\frac{\lvert b-c\rvert}{b+c}$	Yes	Yes	No	Yes	Yes
perfect complex	$\frac{b+c}{2a+b+c}$	$\frac{\lvert b-c\rvert}{2a+b+c}$	Yes	Yes	No	No	Yes

Supplementary Appendix 1: R scripts for the computation of the intersection (*I)* of nestedness and beta diversity and the relative complement (*RC*) of nestedness in beta diversity.

```
#Partitioning of Weiher-Boylen beta diversity using SET partitioning
#Input:
#presence-absence matrix, where rows are sites, columns are species
#Output:
#first distance matrix: Weiher-Boylen beta diversity
#second distance matrix: Intersection of nestedness and beta diversity
#third distance matrix: Relative complement of nestedness in beta diversity
setpart.wb<-function(mat)
{
mat <- as.matrix(mat)
n <- nrow(mat)
mat.b <- ifelse(mat>0, 1, 0)
a <- mat.b %*% t(mat.b)
b <- mat.b %*% (1 - t(mat.b))
c <- (1 - mat.b) %*% t(mat.b)
min.bc <- pmin(b,c)
BD.wb<-(b+c)
I.wb<-matrix(0,n,n)
RC.wb<-matrix(0,n,n)
for (i in 2:n){
      for (j in 1:(i-1)){
      aa=a[i,j];bb=b[i,j];cc=c[i,j]
      if (aa==0) I.wb[i,j]<-0 else I.wb[i,j]<-abs(bb-cc)
      if (aa==0) RC.wb[i,j]<-(bb+cc) else RC.wb[i,j]<-2*min(bb,cc)
}}
res<-list(as.dist(BD.wb),as.dist(I.wb),as.dist(RC.wb))
res
}


#Partitioning of Jaccard dissimilarity using SET partitioning
#Input:
#presence-absence matrix, where rows are sites, columns are species
#Output:
#first distance matrix: Jaccard dissimilarity (beta diversity)
#second distance matrix: Intersection of nestedness and beta diversity
#third distance matrix: Relative complement of nestedness in beta diversity
setpart.j<-function(mat)
{
mat <- as.matrix(mat)
n <- nrow(mat)
mat.b <- ifelse(mat>0, 1, 0)
a <- mat.b %*% t(mat.b)
b <- mat.b %*% (1 - t(mat.b))
c <- (1 - mat.b) %*% t(mat.b)
min.bc <- pmin(b,c)
BD.j<-(b+c)/(a+b+c)
I.j<-matrix(0,n,n)
RC.j<-matrix(0,n,n)
for (i in 2:n){
      for (j in 1:(i-1)){
      aa=a[i,j];bb=b[i,j];cc=c[i,j]
      if (aa==0) I.j[i,j]<-0 else I.j[i,j]<-abs(bb-cc)/(aa+bb+cc)
```

```
      if (aa==0) RC.j[i,j]<-(bb+cc)/(aa+bb+cc) else RC.j[i,j]<-
2*min(bb,cc)/(aa+bb+cc)
}}
res<-list(as.dist(BD.j),as.dist(I.j),as.dist(RC.j))
res
}


#Partitioning of Sørensen dissimilarity using SET partitioning
#Input:
#presence-absence matrix, where rows are sites, columns are species
#Output:
#first distance matrix: Sørensen dissimilarity (beta diversity)
#second distance matrix: Intersection of nestedness and beta diversity
#third distance matrix: Relative complement of nestedness in beta diversity
setpart.s<-function(mat)
{
mat <- as.matrix(mat)
n <- nrow(mat)
mat.b <- ifelse(mat>0, 1, 0)
a <- mat.b %*% t(mat.b)
b <- mat.b %*% (1 - t(mat.b))
c <- (1 - mat.b) %*% t(mat.b)
min.bc <- pmin(b,c)
BD.s<-(b+c)/(2*a+b+c)
I.s<-matrix(0,n,n)
RC.s<-matrix(0,n,n)
for (i in 2:n){
      for (j in 1:(i-1)){
      aa=a[i,j];bb=b[i,j];cc=c[i,j]
      if (aa==0) I.s[i,j]<-0 else I.s[i,j]<-abs(bb-cc)/(2*aa+bb+cc)
      if (aa==0) RC.s[i,j]<-(bb+cc)/(2*aa+bb+cc) else RC.s[i,j]<-
2*min(bb,cc)/(2*aa+bb+cc)
}}
res<-list(as.dist(BD.s),as.dist(I.s),as.dist(RC.s))
res
}
```